Optimal Control for Delayed and Dynamic Networked Control Systems

Hamza Anwar
Department of Electrical and Computer Engineering, Tandon School of Engineering,
New York University, Brooklyn, NY; Email:{ha1082}@nyu.edu

***Abstract*—Prevalent advances in 'Internet of Things' (IoT) has offered interaction of cyber-physical systems for sensing and actuation tasks over the Internet. Cloud computational capabilities have even enhanced such cutting-edge technologies scaling up to critical remote sensing and actuation tasks such as tele-operation, mobile robotics and traffic regulation, promising the future of smart living in smart cities. Internet serves as the communication backbone for these networked systems entailing the advantages of simplicity of use, cost-affordability, unmatched availability and cloud power. The network and control relationship is a core dimension in the design of networked systems and recent trends are in developing co-design solutions to, for instance, the network congestion control scheme and the feedback system's control law design. But, the Internet use for such time-critical systems brings its devils along: packet losses, time-varying time-delays, delay variations and unpredictability of network dynamics. Network analysis are mathematically complex and a natural game-theoretic framework underpins the whole IoT control scheme design problem. We intend to develop a stable congestion control mechanism in Internet-like networks involving dynamic cyber-physical systems under cooperative game-theoretic framework.**

I. Introduction

Over the course of many decades, today's Internet strives to comply with the demands of a broad range of applications. Recent advances in Big Data with a growing computational power and distributed resource usage, have led to highly complicated cyber-physical systems. Geographically distributed cloud servers encapsulate a multitude of potential for remotely controlled networked systems, via the Internet. And to add to all this, the possibility of smart *everything* under the Internet of Things has become an increasingly pressing demand. In such an environment, where we have a variety of different agents interacting via complex networks, real-time physical systems face unexplored challenges. Different agents, that can be devices, sensors, networks, or even whole multi-agent systems, incentivize, enact and interact with other agents, to fulfill their objectives. Such game-theoretic framework underpins the development of schemes in such networks, that avoid the possibility of equilibrium reached by selfish decision-making of all objects. Our work relates congestion control for Internet-style networks with optimal control of real-time cyber-physical systems, and is motivated by the well-known fact that non-cooperative equilibrium is *inefficient*.

Networked control systems (NCS) have been a popular research focus in the industry and academia. The idea behind network control systems is that, many feedback control systems share a common network for communication between sensors, controllers, and plants. Due to various network topologies, each control system might face delays in communication. This leads to delayed sensor output to the controller, and/or delayed control input to the actuators. To design optimal control law, the network effects need to be taken into account. And especially for 'Internet of Things', the complexity of the network is huge involving transfer control protocol (TCP), queuing of packets, buffer size limitations, routing, pricing and even packet drops.

With such considerations, our objective is to design a cooperative congestion control mechanism, that not only optimizes for network utilization and efficiency, but also minimizes control loop cost to meet system performance criterion at the actuator-plant level; which is at stake at the ends of communication channels ('control/network co-design'). We will illustrate the concept with the help of an example. Consider, for instance, a teleoperation example of a doctor who has to perform remote surgery of a patient. The control system's sensors, actuators and plant (the patient) is geographically located at a different place from the controller (the doctor). If the doctor uses the Internet to input control commands for performing the surgery, then the congestion and packet drop delays faced by the operation via the Internet can cause serious damage. At the same time, a smartphone user can make use of the internet for flying a UAV drone around for fun. And so we realize that a common big network should have a congestion control scheme that prioritizes end-users based on the criticality of their applications. In this example, both the end-users are cyber physical systems in themselves, and both have remote controlling agents but a difference in criticality causes asymmetric risks. Criticality is one of the many control system variables. The control system variables are embedded inside the control objective to be optimized by the controller.

We aim to develop congestion control scheme for Internet-style networks of dynamic cyber-physical systems having delays. We model a user's utility function based on its control objective. In standard literature on congestion control over the internet, a time-diminishing log-like utility function is used. We want each users' utility to be a direct consequence of his control objective (assuming that a user here represents a control system). So, for a Linear Quadratic Regulator (LQR) problem that regulates room temperature using sensor data from the Internet, we can assume the utility function of this end-user to be proportionate to the quadratic cost functional it minimizes. But at the same time, it should be noted that the LQR cost itself is depends on the delay caused by the network, for sensor data to reach the controller. This makes our problem a two-stage game problem in extensive form, where the strategies of players at the network level affect strategies of players are the controller level. This two-stage game is solved naturally by a backward induction approach when the players at first stage minimizes their cost which is based on their corresponding second-stage objective functions. We make use of the formulation developed by Alpcan et. al. [1] for the congestion control problem, and the analysis of Linear Quadratic Regulator (LQR) optimal control design with delayed control input by Fridman [3].

A. Related Work

Literature is available on Networked Control Systems (NCS) that takes the effect of network delay into account while designing optimal control strategies. Activity in networked systems is usually characterized, most widely, into two tasks: the control of networks and the control over networks. Control of networks deals with scheduling, routing, flow control, power control and various other resource allocation problems. Objective, thereby, is to utilize the network resources efficiently and fairly in order to provide Quality of Service (QoS) to the end-users. Control over networks deals with control law design given sensor and actuator dynamics while also considering time-varying time-delays, finite capacity, and other network effects into account [6]. Classical approaches in NCS literature try to adapt the control design to a previously defined network. Most researches have modeled the network delay as a random variable focusing on delay prediction and detection problems. They have assumed the network itself as a passive delay-causing block in their block-level control system realization [4][7].

Modern techniques have emerged that talk about co-design and try to model and integrate the network in the control system. For IP-based shared networks, such as the Internet, there is work available in this context, such as [2]. Control parameters to the network, such as transmission rate, packet size and packet structure, are studied in [2], but, the task of congestion control and design of an adaptive pricing mechanism is not co-designed with the control system. We intend to work on a game-theoretic congestion control scheme in Internet-like networks involving dynamic cyber-physical systems.

II. Problem Formulation

A. System Overview

Consider a number of controller-plant subsystems, see Figure 1, each having the following dynamics:

$$\dot{z}(t) = Az(t) + Bu(t - D_i); \quad D_i = \Sigma_{l \in R_i} d_l \qquad (1)$$

where, D_i is the delay in control input for the i-th subsystem and is expressed as the sum of all the delays caused at each intermediate link l that the i-th subsystem uses from the network. We assume that each link can be used by multiple users at the same time given any network topology, but each controllers corresponds to a single plant that he must control.

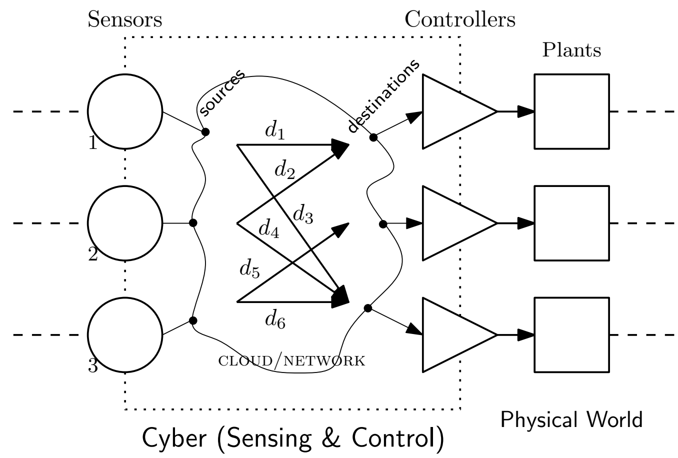

Fig. 1. Depiction of a our framework showing a six-user network (possibly internet) in-between the sensors and controllers, whereas each controller corresponds to a single plant that he must control directly.

Due to the network dynamics, each source-destination pair faces queueing delays (congestion). Even though each control system can have different system matrices and state vectors, but to keep the notation simple, we'll not use subscripts with the control system variables and cost.

We are interested in the design of an optimal control law for each given plant, while also controlling congestion by designing a pricing scheme for the network.

B. Network Model

Without restricting to a wired or a wireless network, we adopt the same network model and pricing scheme for our network as done by the authors of [1]. It is based on fluid approximations. The topology is characterized by N nodes and L links connecting the nodes. There are M users in total, and each user is represented by a unique source-destination pair. For our purpose, the destinations in our framework correspond uniquely to a feedback control system. As for the sources, our sources are sensors. Our framework can be generalized to incorporate multiple sensors transmitting to a single destination. In that case, the only change in our proceeding analysis would be to sum up the delays incurred by all sensors (i.e. $\sum_{i \in \mathcal{M}_j} D_i$ where $\mathcal{M}_j$ denotes the set of users that have the same destination node as the j-th user's destination node). However, we shall avoid extravagant notation usage and the reader to refer to the network model articulated in [1].

- Flow rate at l-th link is the sum of flow rates of all its users (R_i denotes the subset of the set of links corresponding to the route (path) used by i-th user for data transmission):
$$\bar{x}_l = \Sigma_{i: l \in R_i} x_i$$
- Buffer at each link fills up proportional to the link flow rate, $\bar{x}_l$ minus the link capacity C_l:
$$\dot{b}_l(t) = \bar{x}_l(t) - C_l; \text{ if } b_l(t) > 0$$
For simplicity we ignore boundary effects.

- The routing matrix $\mathbf{A}$ defines the capacity constraint (where $A_{l,i}$ is 1 if i uses link l, and is 0 if not):

$$\mathbf{A}\mathbf{x}(t) - \dot{\mathbf{b}}(t) \leq \mathbf{C}$$

- Delay incurred at each link is a non-linear function of flow:

$$\dot{d}_l(t) = \frac{1}{C_l}(\bar{x}_l(t) - C_l);\ \text{if } d_l(t) > 0$$

- End-to-end delay incurred by i-th user (D_i) is the sum of delays at all intermediary links:

$$D_i(t) = \sum_{l \in R_i} d_l(t)$$

  This end-to-end delay is the same delay with which the control input enters the control system ahead, see (1).
- The cost function for each user is composed of a price that he pays for using the network (which itself is bilinear in flow and delay) and his utility function that he wants to maximize:

$$\tilde{J}_i(\mathbf{x}, t) = \alpha_i D_i(t) x_i - U_i(\mathbf{x})$$

  where α_i are pricing parameters.

The goal is to devise a congestion control and pricing scheme by making use of variations in RTT (round-trip-time). Utility of the users is assumed to be strictly increasing, differentiable and strictly concave in the original paper. Our aim here is to design a utility function based on the control system cost that the controller will aim to optimize (locally i.e. given it a delay), and then show it to construct a stabilized network congestion control scheme. A simple dynamic model of the network game is to assume that each user changes its flow rate in proportion to change in the gradient of his incurred cost:

$$\dot{x}_i = -\partial \tilde{J}_i(\mathbf{x}) / \partial x_i$$

Thus, the generalied system becomes:

$$\begin{aligned} \dot{x}_i(t) &= \frac{\partial U_i(\mathbf{x})}{\partial x_i} - \alpha_i D_i(t), \quad i \in \{1 \dots M\} \\ \dot{d}_l(t) &= \frac{\bar{x}_l}{C_l} - 1. \quad l \in \{1 \dots L\} \end{aligned} \tag{2}$$

where the effect of i-th user's flow rate on delay he experiences D_i is ignored. We might need to lift this assumption due to reasons that will become apparent later. Nonetheless, the above framework is constructed here to conclude that the behavior of utility as a function of flow rate will determine system's stability and congestion control scheme, and our proceeding analysis will only tackle this point. Authors have developed a pricing scheme to stabilize this system but they require the utility function of each user to be strictly increasing and strictly concave, or in other words,

$$\frac{\partial U_i(\mathbf{x})}{\partial x_i} \text{ should be decreasing in } x_i$$

An important assumption for such non-linear system is that we operate and propose our results only near our equilibrium point. This is natural to such systems, and considering the Internet's massiveness, it is considerably reasonable. Note that this assumption is critical in our work when establishing the relationship between end-to-end delay faced by user and its flow rate, which later supports our choice of the user's utility function.

C. Control Cost

After having introduced the model and framework, we divert the reader's attention to the behavior of the controller. We require that the controller optimizes for the Linear Quadratic Regulator control objective. Derivation in this section is owe to Fridman [3]. Recall that the destination nodes provide delayed input to the controllers, who minimize a quadratic cost given by

$$J = \int_0^\infty z^T(t) Q z(t) + u^T(t - D_i) R u(t - D_i)\ dt$$

where, $Q \geq 0$ and $R > 0$. Given a delay D_i, the control objective for each control system, is to minimize the above LQR cost functional subject to the linear dynamic constraint of (1). The utility of i-th user of the network will be proportional to this cost as we shall discover later. Here we assume that $u(s) = 0,\ s \in [-D_i, 0)$ and $z(0) = z_0$, and (1) reduces to:

$$\begin{aligned} &\dot{z}(t) = Az(t); \quad z(0) = z_0; \quad t \in [0, D_i), \\ &\dot{z}(t) = Az(t) + Bu(t - D_i); \quad z(D_i) = e^{AD_i} z_0; \quad t \geq D_i. \end{aligned} \tag{3}$$

Assuming $v(t) = u(t - D_i)$, the cost functional (or control cost) can be decomposed into two parts:

$$J = z_0^T \left(\int_0^{D_i} e^{A^T t} Q e^{At} dt \right) z_0 + J_{D_i}$$

$$\text{where } J_{D_i} = \int_{D_i}^\infty [z^T(t) Q z(t) + v^T(t) R v(t)] dt.$$

Minimizing J subject to (3) is reduced to minimization of J_{D_i} for the non-delay system:

$$\dot{z}(t) = Az(t) + Bv(t); \quad z(D_i) = e^{AD_i} z_0; \quad t \geq D_i,$$

which is the standard LQR problem giving the unique solution (if (A, B) is stabilizable and $(A, \sqrt{Q})$ is detectable, that we've assumed):

$$v(t) = -R^{-1} B^T P z(t)$$

where $P \geq 0$ is solution to Riccati equation at steady state, $A^T P + PA - PBR^{-1} B^T P + Q = 0$ The minimal value of $J^*_{D_i} = z(D_i) P z(D_i)$. The optimal control cost, given delay D_i is expressed in the following form:

$$J^*(D_i) = z_0^T \left(\int_0^{D_i} e^{A^T t} Q e^{At} dt \right) z_0 + z_0^T e^{A^T D_i} P e^{AD_i} z_0.$$

In Figures 2, 3 and 4, we've shown various forms of this cost functional as a function of free parameter D_i, the control

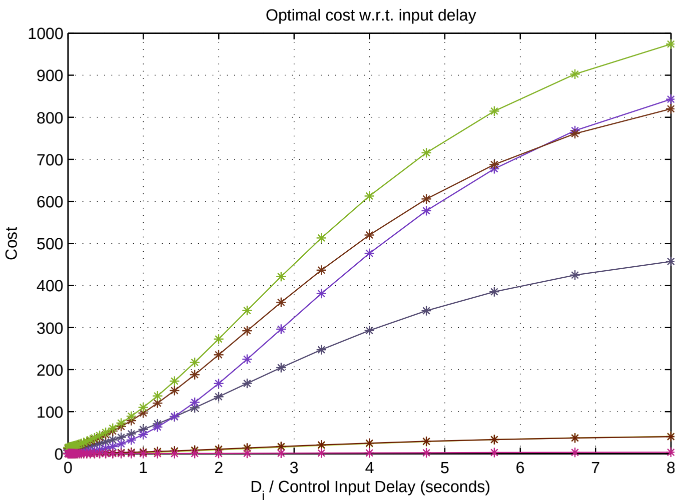

Fig. 2. Optimal control cost as a function of the control input delay, for various values of initial state vector z_0.

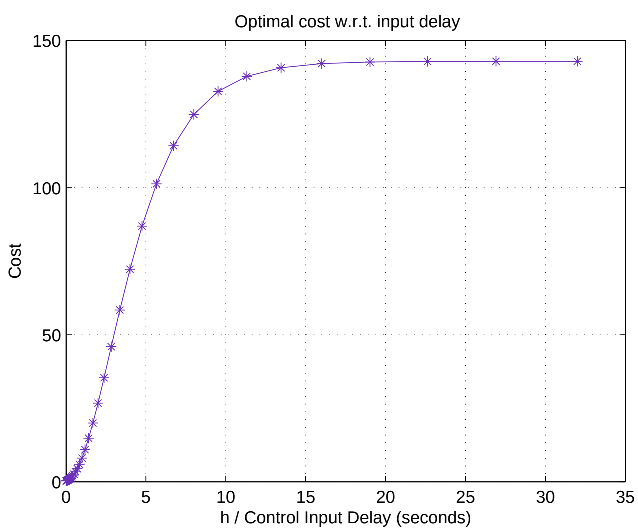

Fig. 3. Zoomed out version of the optimal control cost as a function of control input delay, showing that for very high delays, it settles to a constant (this means that for large delays the initial state vector z_0 falls nearly in the null space of the matrix exponential e^{AD_i}.

input delay for an example fifth-order dynamic control system given by:

$$A = \begin{bmatrix} 0.2 & 0.5 & 0 & 0 & 0 \\ 0 & -0.5 & 1.6 & 0 & 0 \\ 0 & 0 & -14.3 & 85.8 & 0 \\ 0 & 0 & 0 & -33.3 & 100 \\ 0 & 0 & 0 & 0 & -10 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 30 \end{bmatrix}$$

$$R = 1, \;\; Q = diag([1, 0, 0, 0, 0])$$

III. Main Results

A. User's Utility Function

With the explained framework, we want to choose i-th players utility to be inversely related to the control system's

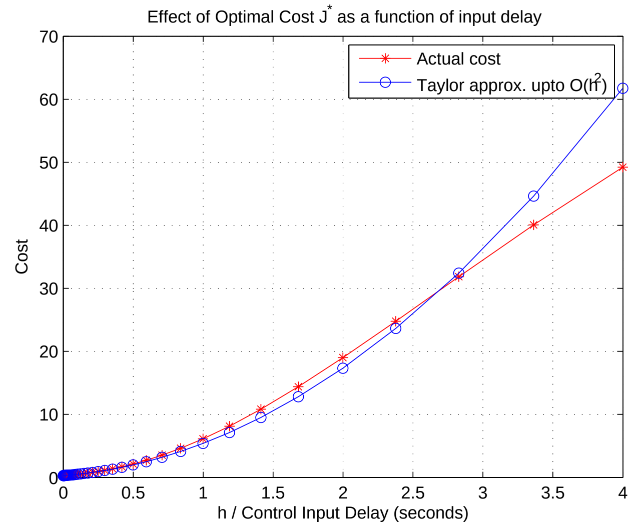

Fig. 4. Taylor Approximation of the optimal control cost as a function of the control input delay variable 'h': $J^* = z_0^T[P + D_i(Q + PA + A^TP) + \frac{D_i^2}{2}(A^T(Q + PA + A^TP) + (Q + PA + A^TP)A) + O(D_i^2)]z_0$.

optimal cost. One possible choice, on which we shall base our analysis is to take it equal to the negative of the control cost:

$$U_i(\mathbf{x}) = -J^*(D_i)$$

The form of the network dynamic equation (2) suggests that we take interest only in the first derivative of cost w.r.t. flow:

$$\frac{\partial J^*}{\partial x_i} = \frac{\partial J^*}{\partial D_i}\frac{\partial D_i}{\partial x_i}$$

However, this is a hard problem because firstly, J^* itself is hard to express and then optimize w.r.t. delay D_i, and secondly, the delay w.r.t. flow rate is also not simple to characterize.

B. End-to-end Delay with-respect-to Flow Rate

Modeling the end-to-end delay with respect to flow rate is difficult owing to the network model. Authors of [1] assume that the gradient of delay w.r.t. flow is always zero. We can not proceed with this consideration because otherwise our user utility function vanishes. However, for large networks, this is a valid assumption. Using (2), we see that i-th players delay is related to the i-th players flow by the following:

$$D_i(t) = (\sum_{l \in R_i} \frac{1}{C_l}) \int_0^t x_i(\tau) d\tau + \int_0^t \sum_{l \in R_i} (\sum_{j: l \in R_j} \frac{x_j}{C_l} - 1) d\tau.$$

For a fluid dynamic system, the instantaneous perturbation of flow rate by one of the users when the system was initially at equilibrium, is an interesting phenomenon. As a valid initial condition of the system, we assume the network is at equilibrium. So, owing to fluid dynamics at equilibrium, we take,

$$\frac{\partial D_i}{\partial x_i} = -\beta(x_i), \quad \text{where } \beta(x_i) \geq 0. \tag{4}$$

where, we haven't assumed any form of the the function β.

Equation (4) means that slight increase in player i's flow rate, reduces its underlying end-to-end delay, only instantaneously. Such an argument makes sense because the fluid dynamics work in this way: an instantaneous increase in a senders flow rate, will jerk the buffer and more packets of this sender will land in the buffer compared to the previous equilibrium allocations. This will instantaneously reduce flow rates of all users that were using the same links. The effect will be propagated and this user will face shorter instantaneous end-to-end delay. But later on, a new equilibrium might be established at different flow rates. This behavior is analogous to a water canal system having many channels and junctions with water dynamics.

C. Optimal Control Cost with-respect-to Delay

Now, we have the following form of utility function's gradient:

$$\frac{\partial U_i}{\partial x_i} = -\frac{\partial J^*}{\partial x_i} = -\frac{\partial J^*}{\partial D_i}\frac{\partial D_i}{\partial x_i} = \beta(x_i)\frac{\partial J^*}{\partial D_i}$$

We proceed further to show that control cost J^* w.r.t. delay is strictly increasing. The gradient is evaluated as follows:

$$\begin{aligned}\frac{\partial J^*}{\partial D_i} &= \frac{\partial}{\partial D_i}\Big(z_0^T(\int_0^{D_i} e^{A^T t}Qe^{At}dt)z_0 + z_0^T e^{A^T D_i}Pe^{AD_i}z_0\Big)\\ &= z_0^T\Big((e^{A^T D_i}Qe^{AD_i})(1) - (e^{A^T(0)}Qe^{A(0)})(0) +\\ &\int_0^{D_i}\frac{\partial}{\partial D_i}(e^{A^T t}Qe^{At})dt + z_0^T e^{A^T D_i}(A^TP + PA)e^{AD_i}\Big)z_0\\ &= z_0^T e^{A^T D_i}(Q + PA + A^TP)e^{AD_i}z_0.\end{aligned}$$

D. Final Form of Utility Gradient

Because of a quadratic form having a positive definite matrix at center, this gradient is non-negative for all values of D_i. Thus, we have,

$$\begin{aligned}\frac{\partial U_i(\mathbf{x})}{\partial x_i} &= \beta(x_i)\frac{\partial J^*(D_i)}{\partial D_i}\\ &= \beta(x_i)z_0^T e^{A^T D_i}\underbrace{(Q + PA + A^TP)}_{\text{positive semi-definite}}e^{AD_i}z_0 \geq 0.\end{aligned} \tag{5}$$

Equation (5) shows the gradient of utility to be non-negative. So, we have established the i-th user's utility function, U_i, to be monotonically increasing with respect to its flow rate, x_i. For strict concavity of the utility, we require its gradient to be monotonically decreasing. In other words, we require the second-derivative of control cost to be strictly negative. It is given by the following relation:

$$\begin{aligned}\frac{\partial^2 J^*}{\partial D_i^2} = z_0^T e^{A^T D_i}\Big(&A^T(Q + PA + A^TP)\\ &+(Q + PA + A^TP)A\Big)e^{AD_i}z_0.\end{aligned}$$

But this can not be only positive or only negative because of positive eigenvalues (unstable modes) of A, see for example

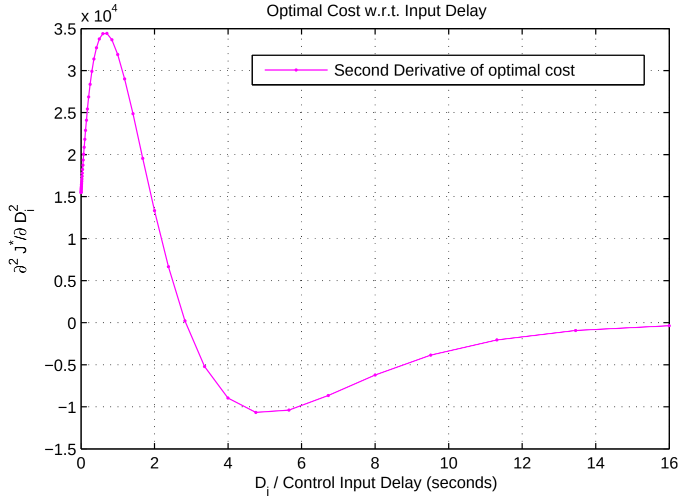

Fig. 5. An example of a control system: Second derivative of its control cost as a function of the control input delay.

Figure 5. With the current choice of utility function, this can only be established if i-th user's delay D_i is exponentially decreasing w.r.t. its flow rate x_i, faster than the exponential increase of control system's cost functional, $J^*(D_i)$, w.r.t. the same delay, D_i. This is hard and plausibly impossible to be certain of.

E. Extension One – Slightly Different Utility Function

In this extension, we propose a different choice of the utility function. Let,

$$U_i(\mathbf{x}) = \gamma\frac{1}{J^*(D_i)}$$

for $\gamma > 0$. This still implies that the gradient is non-negative. However, we have control over the form of the gradient as a function of the flow:

$$\frac{\partial U_i(\mathbf{x})}{\partial x_i} = \gamma\beta(x_i)\frac{1}{[J^*]^2}\frac{\partial J^*}{\partial D_i}.$$

The parameter γ can be adjusted to make this gradient strictly decreasing in flow rate, and with $J^*(D_i)$ in the denominator, the exponential increase of control cost gradient is damped.

There can be other possible choices of utility as well to regularize it into desired form, and this can be explored in future.

F. Extension Two – Time-Varying Input Delay

Up till now, we have assumed that the time-variation in the end-user delay is fast enough, and the network equilibrium is attained so rapidly that the physical control system sees it as a constant delay of input. Works in Networked Control Systems, along with other domains, have taken into account the possibility of a time-varying delay and its effects on the optimal control law design for the control cost, LQR cost in our case. Krstic in [5] has shown that with certain conditions on the control input delay and delay-rate, exponential stability of closed loop control system can be achieved. A similar approach taken by Fridman in [3], but with involved Linear Matrix Inequalities (LMIs), has shown Lyapunov stability for a more general setting of time-varying control input delay. For our problem at hand, the conditions highlighted in [5] are noteworthy:

- Delay itself be uniformly bounded from above, which already holds for us because, $d_l(t) \leq d_{l,max}$.
- Delay be strictly positive (to ensure that the state-space of input dynamics can be defined – this obviously holds).
- Rate of delay be uniformly bounded from below i.e. it can not decrease arbitrarily fast. This requires

$$\begin{aligned} \dot{b}_l(t) &\geq -C_l \quad \forall t \\ \implies \bar{x}_l(t) &\geq 0 \end{aligned}$$

- Rate of delay be bounded from above by one, i.e. $\dot{D}_i(t) < 1 \ \forall t$. For this to hold, we require:

$$\begin{aligned} \dot{b}_l(t) &< C_l \quad \forall t \\ \implies \bar{x}_l(t) &< 2C_l \end{aligned}$$

The last two conditions pose a limit on the flow rate. Both of them can hold because we have already assumed a minimum and a maximum for the flow rates: recall,

$$0 \leq x_i \leq x_{i,max}.$$

IV. Discussion and Conclusion

We have presented a viable control/network co-design scheme for a networked control system. Nash equilibrium attained while considering both the network congestion and optimal control design problems independently, would result in an inefficient design. By incorporating the control objective into a game-theoretic network-congestion-control framework, we've established a two-stage game and, in essence, solved it by backward induction. This marks our notable contribution. Further more, once the stability of the utility function is shown with certainty, we can argue for the whole system to be a two-stage cooperative game (extensive-form), in which the first stage itself is an N-person non-cooperative game (among the users of the network): non-cooperative because any given player's strategy i.e. its flow rate, is independent of the strategies of other players.

For future consideration, we plan to simulate our system using Simulink (or a comparable control system simulation toolbox) joint with a Network Simulator (such as NS-2). To show stability of the proposed mechanism, strict concavity of the utility function is essential. For that, we can study variations of the utility function in-line with the proposed one. Or we can study bounds on the nature of the utility function, for instance, the second derivative of control cost which we require to be strictly negative w.r.t. delay is given as:

$$\begin{aligned} \frac{\partial^2 J^*}{\partial D_i^2} = &\Big\{ z_0^T e^{A^T D_i}\Big(A^T(Q+PA+A^TP)+(Q+PA+ \\ &\qquad A^TP)A\Big)e^{AD_i}z_0. \\ \leq &\Big\{ \lambda_{\max}\Big(A^T(Q+PA+A^TP)+(Q+PA+ \\ &\qquad A^TP)A\Big)||e^{AD_i}z_0||_2^2. \end{aligned}$$

Acknowledgments

This work has been done under Professor Quanyan Zhu, as part of a course project for the course EL-GY 9213 Game Theory for Multi-Agent Systems, Spring 2016 at NYU Tandon School of Engineering.

References

[1] Tansu Alpcan and Tamer Başar. A globally stable adaptive congestion control scheme for internet-style networks with delay. *Networking, IEEE/ACM Transactions on*, 13(6):1261–1274, 2005.

[2] Miguel Díaz-Cacho Medina, Emma Delgado Romero, and Antonio Barreiro Blas. Control/network codesign basics for ip-based shared networks. *Mathematical Problems in Engineering*, 2011, 2011.

[3] Emilia Fridman. *Introduction to time-delay systems: Analysis and control*. Springer, 2014.

[4] Rachana Ashok Gupta and Mo-Yuen Chow. Networked control system: overview and research trends. *Industrial Electronics, IEEE Transactions on*, 57(7):2527–2535, 2010.

[5] Miroslav Krstic. Lyapunov stability of linear predictor feedback for time-varying input delay. *Automatic Control, IEEE Transactions on*, 55(2):554–559, 2010.

[6] Sandro Zampieri. Trends in networked control systems. pages 2886–2894, 2008.

[7] Liqian Zhang, Yang Shi, Tongwen Chen, and Biao Huang. A new method for stabilization of networked control systems with random delays. *Automatic Control, IEEE Transactions on*, 50(8):1177–1181, 2005.